

Exercises for Macroeconomics 1.1

Problem Set 1

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1 Problem 1

Consider a two-period, endowment model of a closed economy, populated by a large and constant number of identical individuals. The representative agent maximizes lifetime utility

$$U(C_1, C_2) = \ln C_1 + \beta \ln C_2, \quad 0 < \beta < 1 \quad (1)$$

subject to the budget constraints

$$\begin{aligned} B_1 &= Y_1 - C_1 \\ B_2 &= (1 + r)B_1 + Y_2 - C_2, \end{aligned}$$

where β is the time-preference factor, C_t is consumption in period t ($= 1, 2$) of the agent's life, Y_t is the endowment of the single, perishable good existing in the economy. In period $t = 1$, individuals may lend/borrow by purchasing/selling one-period real bonds B in the domestic capital market. The real interest rate on these bonds is r .

- Derive the lifetime budget constraint faced by the representative agent. Explain what you assume about B_2 .
- Compute the closed-economy (or "autarky") equilibrium value r^a of the domestic interest rate. Explain why r^a turns out to be an increasing function of the ratio Y_2/Y_1 ("endowment growth"), and a decreasing function of the time-preference factor.
- Introduce a government, whose consumption per capita, G_t , may be financed by lump-sum taxes, T , or by issuing government bonds, B^G . Explain what is meant by "Ricardian Equivalence", and demonstrate that it holds in the present model (your "proof" should include no more than two equations).

- d. Return, for simplicity, to the no-government case analyzed before, and assume that the country studied so far (the "Home" economy) is allowed to trade with the only other country existing in the world (the "Foreign" economy). Home and Foreign's representative agents have identical preferences (as summarized by (1) for both), and may engage in international borrowing and lending. Marking by asterisks the variables pertaining to Foreign (so that, for instance, the foreign representative agent receives exogenously determined endowments Y_1^* and Y_2^*), compute the world equilibrium interest rate, r^W .
- e. Write down the definition of the date $t = 1$ current account for Home. Using this definition, and your answer to question (d), show that Home runs a current account surplus on date 1 if, and only if, $(Y_2/Y_1) < (Y_2^*/Y_1^*)$. Explain.

2 Problem 2

Assume date 2 Home output is a strictly concave function of the capital stock in place multiplied by a productivity parameter,

Home production function $Y_2 = A_2 K_2^\alpha$

($\alpha < 1$), with a parallel production function in Foreign. Date 1 outputs are exogenous because they depend on inherited capital stock (state variable).

- Investment is determined so that the marginal product of capital equals r . Show that this equality implies $K_2 = \left(\frac{\alpha A_2}{r}\right)^{\frac{1}{1-\alpha}}$.
- Show that Home's II schedule can be written as

$$I(r) = K_2 - K_1 = \left(\frac{\alpha A_2}{r}\right)^{\frac{1}{1-\alpha}} - K_1$$

- Derive Home's date 1 consumption function, and show that it can be written as

$$C_1(r) = \frac{1}{1+\beta} \left(Y_1 - I_1 + \frac{Y_2 - I_2}{1+r} \right)$$

- Using $I_2 = -K_2$ (negative investment, all capital stock will be eaten in period 2) and the result in 1 - 3 explain why

$$C_1(r) = \frac{1}{1+\beta} \left[K_1 + Y_1 + \frac{(1+\alpha)}{1+r} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} \right]$$

and conclude that the equation for SS is the upward sloping curve

$$S_1(r) = Y_1 - C_1(r) = Y_1 - \frac{1}{1+\beta} \left(Y_1 + K_1 + \left(\frac{1-\alpha}{1+r}\right) \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} \right)$$