Exercises for Macreconomics 1.1
Problem Set 2

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1 Problem 1, OLG (1)

Suppose that N_t 2-period-lived (identical) individuals are born in period t and that $N_{t+1} = (1+n) N_t$. For simplicity, let utility be logarithmic: $U_t = \ln(C_{1,t}) + \beta \ln(C_{2,t+1})$. A subscript 1 (2) denotes that the variable pertains to the young (old) generation. The economy is closed and capital markets are perfect. Each individual is endowed with Y_1 when young and with Y_2 when old.

1. Show that the number of young alive is a constant fraction of total population.

2. The utility function is concave, the household, therefore, prefers to smooth consumption. In this model there are four possible ways the household could smooth consumption (one more than in the Fisher model). What are these?

3. From now on we will assume that the good is not storable and that the economy is closed. Will there be trade within the same generation? Will there be trade between generations?

4. In a decentralized economy, write down the household's budget constraint and derive the equilibrium interest rate in terms of exogenous variables.

5. Show the decentralized equilibrium graphically in the C_1, C_2 space. In particular, show the consumption point and the endowment point. Note that the economy is stationary, i.e. consumption of the old or of the young is the same regardless of the generation. C_2 (on the vertical axis), therefore, stands for both the consumption of the old in t and of the young in $t+1$ (when they are old).

6. There are situations where the decentralized equilibrium is not pareto optimal. Give an intuition why in the OLG model a social planner can improve the household's welfare and in the (infinite horizon) Fisher model not.

7. Derive the social planner's budget constraint. Hint: total consumption in t must equal total endowment in t.

8. Assume that the social planner can impose a lump sum tax τ (subsidy if negative) to redistribute the consumption good from one generation to another. Per capita consumption of the young then becomes

$$C_1 = Y_1 - \tau$$

and per capita consumption of the old

$$C_2 = Y_2 + \tau (1 + n)$$

The objective of the social planner is to maximize utility subject to feasibility. The choice variable of the social planner is τ. Set up the social planner's maximization problem.

9. Derive the first order condition of the planner's problem and show that $\tau > 0$ if $n > r$.

2 Problem 2, OLG(2)

Suppose that N_t 2-period-lived individuals are born in t and that $N_t = (1 + n) N_{t-1}$. For simplicity, let utility be logarithmic with no discouting: $U_t = \ln (C_{1,t}) + \ln (C_{2,t+1})$. Each individual born at time t is endowed with A units of the economy's single good. The good can either be consumed or stored. Each unit stored yields $x > 0$ units of the good in the following period.

Finally assume that in the initial period, period 0, in addition to the N_0 young individuals each endowed with A units of the good, there are $\frac{1}{1+n} N_0$ individuals who are alive only in period 0. Each of these "old" individual's is endowed with some amount Z of the good; their utility is simply their consumption in the initial period, C_{20}.

1. Describe the decentralized equilibrium of this economy. (Hint: given the overlapping-generations structure, will the members of any generation engage in transactions with members of another generation?

2. Consider paths where the fraction of agent's endowments that is stored, f_t, is constant over time. What is total consumption (that is, consumption of all the young plus consumption of all the old) per person on such a path as a function of f? If $x < 1+n$ what value of f satisfying $0 \leq f \leq 1$ maximizes consumption per person? Is the decentralized equilibrium Pareto-efficient in this case? If not, how can a social planner raise welfare?