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MEc, Macro 2
Problem Set 1

1 Solving rational expectations models

(a) Suppose the variable y_t follows

$$y_t = \alpha_0 + \alpha_1 \mathrm{E}_t y_{t+1} + u_t, \tag{1}$$

where u_t is of moving-average form, so

$$u_t = v_t + \theta v_{t-1}, \tag{2}$$

and where v_t is white noise.

Find a solution for y_t.

(b) Suppose the variable y_t follows

$$y_t = \alpha_0 + \alpha_1 \mathrm{E}_t y_{t+1} + \alpha_2 y_{t-1} + u_t, \tag{3}$$

where u_t is a first-order auto-regressive disturbance, so

$$u_t = \rho u_{t-1} + \xi_t, \tag{4}$$

and where ξ_t is white noise.

Given the conjecture

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 u_{t-1} + \phi_3 \xi_t, \tag{5}$$

characterize the solutions for $\phi_0, \phi_1, \phi_2, \phi_3$. What can we say about ϕ_1?

2 Solving an RBC model

In this problem you are asked to solve a real business cycle model with the method of undetermined coefficients.

A log-linearized version of the RBC model (assuming perfect foresight) is given by

$$\begin{aligned} c_t &= \frac{Y}{C} z_t + \frac{K}{\beta C} k_{t-1} - \frac{K}{C} k_t, && (6) \\ r_t &= [1 - \beta(1-\delta)] [z_t - (1-\rho) k_{t-1}], && (7) \\ 0 &= \eta (c_t - c_{t+1}) + r_{t+1}, && (8) \\ z_t &= \phi z_{t-1} + \varepsilon_t. && (9) \end{aligned}$$

Equation (6) is the resource constraint, equation (7) is the return function, equation (8) is the consumption Euler equation, and equation (9) specifies the disturbance term z.

To solve for the dynamics with the method of undetermined coefficients we need a conjecture. A natural conjecture is to assume that the three endogenous variables (k_t, c_t, r_t) are a function of the exogenous variable z_t and lagged values of the state variable k_t:

$$\begin{aligned} k_t &= v_{kk} k_{t-1} + v_{kz} z_t && (10) \\ r_t &= v_{rk} k_{t-1} + v_{rz} z_t && (11) \\ c_t &= v_{ck} k_{t-1} + v_{cz} z_t, && (12) \end{aligned}$$

where the coefficients $v_{kk}, v_{kz}, v_{rk}, v_{rz}, v_{ck}, v_{cz}$ are to be determined. Note also that

$$\mathrm{E}_t z_{t+1} = \phi z_t \qquad (13)$$

(a) What are the coefficients v_{rk} and v_{rz}?

(b) Show that the coefficient v_{ck} is equal to

$$v_{ck} = \frac{K}{\beta C} - \frac{K}{C} v_{kk}.$$

(c) What is the coefficient v_{cz}? (It is similar to v_{ck}.)

(d) Show that

$$v_{kk} = \frac{v_{ck}}{v_{ck} - v_{rk}/\eta}.$$

(e) Show that

$$v_{kz} = \frac{v_{cz} - v_{cz}\phi + v_{rz}\phi/\eta}{v_{ck} - v_{rk}/\eta}.$$

(f) The coefficient v_{kk} relates the new value of the endogenous state variable k_t to its old value k_{t-1}, so it captures the essence of the dynamics of the system. Show that the solution for v_{kk} can be expressed as

$$0 = v_{kk}^2 - \gamma_1 v_{kk} + \gamma_2.$$

What are γ_1 and γ_2?

(g) Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In exercise (f), what are a, b, and c? Can you say anything about which is the relevant root for our problem?