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MEc, Macro 2
Problem Set 2

1 Money in the utility function

Use the model with money in the utility function discussed in class.

(a) Suppose the utility function is separable in consumption and real money balances:

$$u(c_t, m_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{m_t^{1-\gamma}}{1-\gamma}. \tag{1}$$

Set up the optimization problem of a representative agent, and derive the consumption Euler equation and the money demand equation.

(b) Suppose instead the utility function is non-separable, so

$$u(c_t, m_t) = \frac{1}{1-\gamma}\left[c_t^{\alpha} m_t^{1-\alpha}\right]^{1-\gamma}. \tag{2}$$

Derive the consumption Euler equation and the money demand equation.

(c) How do the consumption Euler equation and the money demand equation differ between the two cases?

(d) In modern monetary policy analysis, it is often assumed that the central bank directly sets the short-term interest rate i_t, and that money is not "essential" to the model, in the sense that changes in m have no separate effect on consumption (and therefore output) other than through the interest rate. Under what conditions on the utility function does this seem a valid assumption?

2 Log-linearization

(a) Use the standard (stochastic) consumption Euler equation

$$1 = \mathrm{E}_t \left\{ \beta R_{t+1} \left[\frac{c_{t+1}}{c_t} \right]^{-\sigma} \right\}, \tag{3}$$

where $R_{t+1} \equiv (1 + i_t)P_t/P_{t+1}$ is the gross real interest rate. Find a log-linear approximation around steady state. Solve for $\hat{c}_t$. What is the interpretation of this equation?

Define the "output gap" x_t as the log deviation of output from its flexible-price level:

$$x_t \equiv \hat{y}_t - \hat{y}_t^f. \tag{4}$$

In a closed economy without government spending and capital accumulation, use the consumption Euler equation to derive an equation that determines the output gap x_t in terms of the expected output gap $\mathrm{E}_t x_{t+1}$, the real interest rate $\hat{\imath}_t - \mathrm{E}_t \pi_{t+1}$, and the disturbance term $\mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f$.

(b) Assume instead that households' utility displays "external habit formation" of the sort

$$u\left(c_t^i\right) = \frac{\left(c_t^i - hc_{t-1}\right)^{1-\sigma}}{1-\sigma}, \tag{5}$$

so the utility of household i depends on its own consumption, c_t^i, relative to past aggregate consumption, c_t, where $0 \leq h < 1$.

Then the consumption Euler equation is

$$1 = \mathrm{E}_t \left\{ \beta R_{t+1} \left[\frac{c_{t+1}^i - hc_t}{c_t^i - hc_{t-1}} \right]^{-\sigma} \right\}. \tag{6}$$

(Optional: Can you show this?)

Assuming that $c_t^i = c_t$ in equilibrium, show that the log-linearized consumption Euler equation is given by

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}\mathrm{E}_t\hat{c}_{t+1} - \frac{1-h}{(1+h)\sigma}\left[\hat{\imath}_t - \mathrm{E}_t\pi_{t+1}\right]. \tag{7}$$