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# MEc, Macro 2
# Problem Set 3

## 1 The Rotemberg (1982) model of price rigidity

Suppose the representative firm $i$ faces a quadratic cost $\gamma(p_t^i - p_{t-1}^i)^2$ of changing its price. Assume that the firm's optimal price in the absence of this cost is $p_t^*$, which follows

$$p_t^* = p_t + \alpha x_t + \varepsilon_t, \tag{1}$$

where $p_t$ is the aggregate price level, $x_t$ is the output gap, $\varepsilon_t$ is a white noise supply shock, and $\alpha > 0$.

When setting its price, the firm is assumed to minimize the expected quadratic deviation of its price $p_t^i$ from the optimal price $p_t^*$ given the cost of price adjustment. Thus the firm sets its price to minimize

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \left(p_t^i - p_t^*\right)^2 + \gamma \left(p_t^i - p_{t-1}^i\right)^2 \right], \tag{2}$$

where $\beta$ is a discount factor.

(a) Derive the first-order condition from the firm's minimization problem.

(b) Assuming that all firms are identical, so $p_t^i = p_t$ in equilibrium, use the first-order condition to derive the New-Keynesian Phillips curve, i.e., a relationship between current inflation, expected inflation, the output gap, and the supply shock.

(c) How does inflation depend on the cost parameter $\gamma$ and the parameter $\alpha$? Explain.

(d) How often do firms change their price in this model? If the model is used to describe quarterly data, is this implication consistent with the empirical evidence on firms' price setting behavior?

## 2 The Taylor (1979) model of wage rigidity

Suppose nominal wages are set for two periods, so the average nominal wage is given by

$$w_t = \frac{\tilde{w}_t + \tilde{w}_{t-1}}{2}, \tag{3}$$

where $\tilde{w}_t$ is the contract wage. Furthermore, assume that when setting their contract wage, workers care about their real wage over the life of the contract and the current state of the economy, so the contract wage is

$$\tilde{w}_t = \frac{1}{2}\left[p_t + \mathrm{E}_t p_{t+1}\right] + \alpha x_t, \tag{4}$$

where $p_t$ is the aggregate price level and $x_t$ is the output gap and $\alpha > 0$.

Finally, prices are assumed to be a markup on marginal cost, and the markup is normalized to zero, so

$$p_t = w_t. \tag{5}$$

(a) Derive a version of the New-Keynesian Phillips curve, i.e., inflation in terms of expected inflation, the output gap, and the error term

$$\theta_t \equiv \mathrm{E}_{t-1} p_t - p_t. \tag{6}$$

Hint: Substitute (4) into (3) and the resulting expression into (5).

(b) How does inflation depend on the parameter $\alpha$? Explain.

(c) Suppose there is a contractionary shock to monetary policy, so the the output gap and therefore inflation unexpectedly fall. What happens to the error term in the Phillips curve? What does this imply for the effects of shocks in this model?

## 3 Monopolistic competition à la Dixit and Stiglitz (1977)

Suppose that the consumption index for household $i$ is equal to

$$C_i = \left[\int_0^1 Z_j^{1/\theta} C_{ij}^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}, \tag{7}$$

where $C_{ij}$ is the individual's consumption of good $j$ and $Z_j$ is a taste shock for good $j$. Suppose the household has real resources $Y_i$ to spend on goods, so the budget constraint is

$$\int_0^1 P_j C_{ij} dj = Y_i. \tag{8}$$

(a) Derive the first-order condition for the household's problem of choosing $C_{ij}$ to maximize $C_i$ subject to the budget constraint. Express demand for good $j$, $C_{ij}$, in terms of $Z_j$, $P_j$, $C_i$, and the Lagrange multiplier on the budget constraint.

(b) Using the corresponding equation for some other good $k$, show that relative demand for the two goods is given by

$$\frac{C_{ij}}{C_{ik}} = \frac{Z_j}{Z_k}\left[\frac{P_j}{P_k}\right]^{-\theta}. \tag{9}$$

(c) Solve this expression for $C_{ik}$ and use the budget constraint to find $C_{ij}$ in terms of $Y_i, Z_j, P_j$, and $\int_0^1 Z_j P_j^{1-\theta} dj$.

(d) Substitute your result in (c) into the definition of $C_i$ and show that

$$C_i = \frac{Y_i}{P}, \tag{10}$$

where

$$P \equiv \left[\int_0^1 Z_j P_j^{1-\theta} dj\right]^{1/(1-\theta)}. \tag{11}$$

This means that $PC_i = Y_i$, so $P$ is the appropriate aggregate price index.

(e) Use the results in (c) and (d) to show that

$$C_{ij} = Z_j \left[\frac{P_j}{P}\right]^{-\theta} C_i. \tag{12}$$

What is the interpretation of this equation? What is the Lagrange multiplier in (a)?

## References

Dixit, Avinash K. and Joseph E. Stiglitz, "Monopolistic competition and optimum product diversity," *American Economic Review* 67 (3), 297–308, June 1977.

Rotemberg, Julio J., "Monopolistic price adjustment and aggregate output," *Review of Economic Studies* 49 (4), 517–531, October 1982.

Taylor, John B., "Staggered wage setting in a macro model," *American Economic Review* 69 (2), 108–113, May 1979.