Ulf Söderström
IEP and IGIER, Università Bocconi

December 10, 2003

MEc, Macro 2
Problem Set 4

1 An optimal simple monetary policy rule

Suppose the economy is characterized by the two equations

$$
\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] + u_t, &(1)\\
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, &(2)
\end{aligned}
$$

where the cost-push shock e_t follows

$$
e_t = \rho e_{t-1} + \varepsilon_t. \tag{3}
$$

Suppose the central bank's loss function is given by

$$
\mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left[\pi_{t+k}^2 + \lambda x_{t+k}^2\right], \tag{4}
$$

and that the central bank can commit to a policy rule of the form

$$
\pi_t = \gamma e_t. \tag{5}
$$

(a) What is the optimal value of γ?

(b) What is the equilibrium output gap under this policy?

(c) Compare this policy to the unconstrained optimal policy under discretion discussed in class. Why is it similar?

2 Monetary policy and shocks

Suppose the economy is described by the following log-linearized system:

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] + \mathrm{E}_t\left[z_{t+1} - z_t\right] + u_t, \tag{6}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{7}$$

where z_t is a productivity shock, u_t is a demand shock, and e_t is a cost-push shock. Assume that these shocks follow

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \tag{8}$$

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \tag{9}$$

$$e_t = \rho_e e_{t-1} + \varepsilon_t^e, \tag{10}$$

where the ε's are white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left[\pi_{t+k}^2 + \lambda x_{t+k}^2\right]. \tag{11}$$

(a) Derive the optimal time-consistent policy. Write down the first-order conditions and the reduced-form solutions for π_t and x_t.

(b) Derive the interest rate rule implied by the optimal policy.

(c) Show that under the optimal policy, the nominal interest rate is increased enough to raise the real interest rate in response to a rise in expected inflation.

(d) How will x_t and π_t move in response to a demand shock? To a productivity shock?

3 A "speed-limit target" for monetary policy

Suppose the economy is described by

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] + u_t, \tag{12}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{13}$$

where the cost-push shock e_t follows

$$e_t = \rho e_{t-1} + \varepsilon_t. \tag{14}$$

(a) Suppose the central bank's loss function is given by

$$\mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left[\pi_{t+k}^2 + \lambda x_{t+k}^2\right]. \tag{15}$$

Derive the first-order conditions for the optimal policy under discretion and commitment.

(b) Suppose instead that the central bank wants to stabilize the change in the output gap rather than the level, so the loss function is

$$\mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left[\pi_{t+k}^2 + \lambda \left(\Delta x_{t+k}\right)^2\right]. \tag{16}$$

Derive the first-order conditions for the optimal policy under discretion.

(c) Suppose society's loss function is given by (15), and the central bank cannot commit to an optimal policy, but must use the time-consistent policy. Would it be better for society if the central bank uses the loss function (15) or (16)? Why?

Reference: Walsh, C. E. (2003). Speed limit policies: The output gap and optimal monetary policy. *American Economic Review*, **93**(1), 265–278

4 An optimally conservative central banker (optional)

In the standard New-Keynesian model, the optimal time-consistent policy implies that the variance of inflation and the output gap are

$$\mathrm{Var}(\pi_t) = \left(\frac{\lambda}{\kappa^2 + \lambda(1-\beta\rho)}\right)^2 \sigma_e^2, \tag{17}$$

$$\mathrm{Var}(x_t) = \left(\frac{\kappa}{\kappa^2 + \lambda(1-\beta\rho)}\right)^2 \sigma_e^2, \tag{18}$$

where λ is the central bank's weight on output stabilization, and σ_e^2 is the variance of the cost-push shock.

Suppose the government delegates monetary policy to an independent central banker with preference λ who chooses the optimal time-consistent policy, while society's loss function is given by

$$\mathrm{Var}(\pi_t) + \lambda^S \mathrm{Var}(x_t), \tag{19}$$

where λ^S is the weight on output stabilization for society as a whole.

If the government can choose the value of the central bank's preference parameter λ, what value would minimize society's loss function, given λ^S? How does this optimal λ depend on the other parameters in the model?